Bibliographie

D. W. Barnes and J. M. Mack, An Algebraic Introduction to Mathematical Logic, V+121 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

According to the authors' intention declared in the Preface, "this book is intended to make mathematical logic available to mathematicians working in other branches of mathematics". Despite of the title the presentation uses very few algebraic means.

Chapter I accumulates some simple notions such as the free algebra, relatively free algebra and variety of universal algebras. Chapters II and III deal with Propositional calculus. Chapter IV develops both syntax and semantics of Predicate calculus and proves Gödel's Completeness Theorem by the method of Henkin. In Chapter V mathematical theories based on the first order predicate calculus are investigated. In particular, the Löwenheim-Skolem Theorem and the elimination of quantifiers are studied. Chapter VI lists the axioms of the Zermelo—Frankel set theory. Chapter VII introduces the notions of ultraproduct, ultrapower and direct limit and there is a nice proof of the theorem that every field has an algebraic closure. Non-standard models are discussed in Chapter VIII with applications to elementary non-standard analysis. In Chapter IX Turing machines and Gödel numbers are introduced to explain the notion of calculability and solvability. In particular, Church's theorem on undecidability of the predicate calculus is included. Finally, Hilbert's Tenth Problem and a brief outline of its solution by Matiyaševič are presented in Chapter X.

The book is very clearly written, supplied with excercises at the end of sections (some of them need far more knowledge than provided by the text).

P. E.-Tóth (Szeged)

B. Bollobás, Graph theory: An Introductory Course (Graduate Texts in Mathematics, Vol. 63), X+180 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

The 8 chapters of the book (Fundamentals; Electrical Networks; Flows, Connectivity and Matching; Extremal Problems; Colouring; Ramsey Theory; Random Graphs; Graphs and Groups) contain gradually more and more involved results, with several relations to other branches of mathematics.

The reviewer was pleasantly surprised to find a full chapter on electrical networks, and feels some competence to criticize this chapter in a more detailed way. The order of the presentation is quite unusual. In other texts, Theorem 1 is presented usually much later than Theorem 7. (However, the other texts are written mainly to students in engineering, while the order in this book seems to be more adequate for mathematicians.) On the other hand, one sees no reason why should the material of §2 separate those in §§1 and 3. The references at the end of the chapter refer to §2 only and the exercises, related to §2 are also more adequate than the rest. Probably a few remarks on electric network duality and its relation to planar graphs could also be in order.

The author successfully meets two contradicting requirements: most of the important branches of the theory are presented; still, several deep results are included. The presentation is clear, there is a strong attempt to present typical methods and ways of reasoning in addition to the results.

A great advantage of the book is the good selection of exercises, containing quite a few unusual and deep results.

The book is a valuable addition to the literature and is highly suggested for students and teachers of graph theory.

A. Recski (Budapest)

Carl de Boor, A Practical Guide to Splines (Applied Mathematical Sciences, 27) XXIV+392 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

The textbook grew out of lectures on splines delivered by the author at Redstone Arsenal in 1976 and at White Sands Missile Range in 1977. It stresses the representation of splines as linear combinations of B-splines, provides proofs only for some of the results stated but offers many Fortran programs. The reader is requested to consult a few books listed in the bibliography if he wishes to develop a more complete picture of spline theory. As the author says in the Preface, his book presents only those parts of spline theory which he found useful in calculations. Indeed, this book is an excellent one for everyone who deals with applied mathematical problems involving polynomial splines.

The following outline may provide an idea of the content. Chapters I and II recapitulate material needed later from the ancient theory of polynomial interpolation. The next four chapters follow somewhat the historical development, with piecewise linear, piecewise cubic, and piecewise parabolic approximation discussed. The computational handling of piecewise polynomial functions is the subject of Chapters VII and VIII. B-splines are introduced in Ch. IX, while Chs. X and XI are intended to familiarize the reader with them. The remaining chapters contain various applications, all involving B-splines: the smoothing spline and least-squares spline approximation for noisy data, the use of splines in solving differential equations, approximation of curves etc. The final chapter treat with the simplest generalization of splines to several variables.

Each chapter ends with some problems to test the reader's understanding of the material, to bring in additional material and to urge numerical experimentation with the programs provided. The Bibliography does not claim completeness, it contains only items referred to in the text. For the reader's convenience a Postscript on Things not Covered, a List of Fortran Programs, and a Subject Index complete the book.

F. Móricz (Szeged)

Yuan Shih Chow and Henry Teicher, Probability Theory (Independence, Interchangeability, Martingales), XV+455 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

The concern of this book is the measure theoretical foundations of probability theory and the major theorems of the subject. The main topics treated are independence, interchangeability and martingales as indicated in the title. Thus, such important concepts as Markov and stationary processes are not even defined, although the special cases of sums of independent random variables and interchangeable random variables are dealt with extensively. Likewise, continuous parameter stochastic processes, although alluded to, are not discussed.

The book is intended to serve as a graduate text in probability theory. No knowledge of measure or probability is presupposed. A novel feature is the attempt to intertwine measure and probability rather than, as customary, to set up between them a sharp demarkation. Particular emphasis is placed upon stopping times, on the one hand, as tools in proving theorems, and on the other, as objects of interest in themselves. For example, optimal stopping problem, limit destributions of sequences of stopping rules (i.e. finite stopping times), randomly stopped sums are of special interest. Many of the proofs given and a few of the results are new. Occasionally, a classical notion is looked at through new lenses (e.g. reformulation of the Lindeberg condition).

Chapter 1—3 contain the elements of measure theory, binomial random variables and independence involving the Borel—Cantelli theorem and Kolmogorov zero-one law. It is surprising how much probability can be developed without even a mention of integration. A number of topics treated later in generality are foreshadowed in the very tractable binomial case. Ch. 4 is devoted to integration in a probability space, while Ch. 6 to measure extensions, Lebesgue—Stieltjes measure and the Kolmogorov consistency theorem.

Readers familiar with measure theory can plunge into Ch. 5 after reading Section 3.2. A one-year course presupposing measure theory can be built around Chapters 5, 7, 8, 9, 10, 11 and 12. In more detail, Ch. 5 treats the sums of independent random variables, Ch. 7 introduces the notions of conditional expectation, conditional independence, and martingales. Ch. 8 deals with distribution functions and characteristic functions, involving the Fréchet—Shohat, Glivenko—Cantelli and Cramér—Lévy theorems. The central limit theorems are studied for the independent case, interchangeable case and martingale case (Ch. 9), while the laws of large numbers, the law of the iterated logarithm for independent case (Ch. 10), Martingales are introduced in Section 7.4, where the upward case is treated, and then developed more generally in Ch. 11. The final Ch. 12 contains material concerning infinite divisible laws.

The book is complemented by a List of Abbreviations, a List of Symbols and Conventions, and an (author and subject) Index. Each section ends with exercises, and each chapter with references. The exercises are used to extend theory, to illustrate a theorem, or to obtain a classical result from one recently proven.

The presentation is self-contained and unified. It is highly recommended for every graduate student or mathematician who wishes to begin studies in Probability Theory.

F. Móricz (Szeged)

Combinatorial Mathematics. VI, Proceedings of the Sixth Australian Conference on Combinatorial Mathematics, Armidale, August 1978. Edited by A. F. Horadam and W. D. Wallis (Lecture Notes in Mathematics, Vol. 748), IX+206 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

The volume contains texts of three of the invited addresses (R. B. Eggleton and D. A. Holton on graphic sequences, S. O. Macdonald on the interaction between combinatorics and graph theory, B. D. McKay and R. G. Stanton on generalized Moore-graphs) and 15 contributed papers (about 40—40% of which refer to designs and graphs, respectively).

A. Recski (Budapest)

George Grätzer, Universal Algebra, 2nd edition, XVIII+581 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

The first edition of Grätzer's Universal Algebra came out in 1968 and instantly became *the* reference book of its topic. The very successful choice of the material is testified by the fact that, after eleven years and about a thousand new articles in the area, a second edition containing the unchanged text of the first one has been justified and necessitated.

Clearly, in order to remain the reference book also in the future, this second edition has had to mirror the rapid development of universal algebra in the seventies. For this aim, it contains seven appendices, partly written by invited experts, and an abundant additional bibliography which includes even several important articles not in print yet.

The first appendix (Shortly: A1) is a survey of recent research not covered in the further appendices. A2 is a review of the solved problems, posed in the first edition. A3 (by B. Jónsson) introduces into Mal'cev conditions and congruence varieties, A4 (by W. Taylor) surveys equational theories, A5 (by R. N. Quackenbush) gives a picture on primal algebras and other generalizations of Boolean algebras, and A6 (by G. H. Wenzel) deals with equational compactness. Finally, A7 contains the proof of a deep new result of the author and W. A. Lampe, namely, that the congruence lattice, the subalgebra lattice and the automophism group are independent for infinitary algebras.

The time-tested basic text with these nicely written mini-monographs added will serve, no doubt, as the standard universal algebra book, in the eighties, too.

B. Csákány (Szeged)

S. W. Hawking and W. Israel, editors, **General Relativity. An Einstein Centenary Survey,** XV + 920 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1979.

The Einstein centenary arose widerspread new interest for general relativity all over the world. The present book is a most appropriate commemoration on Einstein's hundredth birthday. Twenty-one of the world's leading relativists collaborated on this survey and gave a render of the current state of research.

The book starts with an introductory survey of S. W. Hawking and W. Israel. The papers of C. M. Will, D. H. Douglass and V. B. Braginsky deal with the confrontation between gravitation theory and experiments. The work of A. E. Fischer and J. E. Marsden discusses the initial value problem and the Cauchy problem for relativity and they give the dynamical formulation of general relativity too.

The discovery of exotic astronomical objects (quasars, pulsars, and X-ray sources) necessitated the development of theories which can explain the complex behaviour of these objects theoretically. Such are the theory of cosmology, black hole physics, theory of singularity, the early history of the universe, e.t.c. A lot of papers discuss these fields by the authorities who, strictly speaking, created these theories. We can mention here, e.g., the following names: R. Gerock and G. T. Horowith (Global structure of spacetimes), B. Carter (The general theory of the mechanical, electromagnetic and thermodynamic properties of black holes), S. Chandrasekhar (An introduction to the theory of the Kerr metric and its perturbations), R. D. Blanford and K. S. Thorne (Black hole astrophysics), R. H. Dicke and P. J. E. Peebles (The big bang cosmology—enigmas and nostrums), Ya. B. Zel'dovich (Cosmology and the early universe), M. A. H. MacCallum (Anisotropic and inhomogeneous relativistic cosmologies), R. Penrose (Singularities and time-asymmetry).

One of the most exciting problem of physics is the unification of general relativity with quantization and with other laws of physics. The book treats the present status of this field with an abundant and profound material. C. W. Gibbons surveys the present quantum field theory in curved spacetime. B. S. DeWitt gives a new synthesis of quantum gravity. The article of S. W. Hawking shows how the path integral approach can be applied to the quantization of gravity and how it leads to the concepts of black hole temperature and intrinsic quantum mechanical entropy. In the last article of the book S. Weinberg deals, in connection with ultraviolet divergences in quantum gravity, with the future of quantum gravity and gives several conjectures concerning the evolution of the quantization in relativity.

We want to emphasize also the highly intelligent editorial work and the very nice appearance of the book. The editors should be congratulated for presenting us a work which will remain the "Bible" of relativity for many decades to come.

Z. I. Szabó (Szeged)

H Hermes, Introduction to Mathematical Logic, XI+242 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1973.

This volume is a valuable introductory text in the classical two-valued predicate logic.

After three editions in German, the original text was translated into English by D. Schmidt. Concerning the material covered is no difference between the English and the third German edition.

Both syntactical and semantical approaches are developed with a little more emphasis on the latter. After an excellent introduction the language and calculus of the first-order predicate logic are given in Chs. II—IV. The treatment leads to the Gödel's Completeness Theorem in Ch. V. In Ch. VI, the axiomatic number theory and the second order predicate logic are introduced, on making the notion of completeness clearer. Ch. VIII includes pure model-theoretic proofs of some basic results in definition theory (such as theorems of Robinson, Craig, Beth, etc).

In the remaining chapters (VII and IX) useful techniques are presented to derive some well-known logical connectives and normal forms. A systematic treatment of substitution is also included here.

P. E.-Tóth (Szeged)

Joram Lindenstrauss and Lior Tzafriri, Classical Banach Spaces. II (Function Spaces) (Ergebnisse der Mathematik und ihrer Grenzgebiete, 97), X+243 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

The second volume on classical Banach spaces by the same authors [Volume I: *Classical Banach Spaces*. I *(Squence spaces)*, Springer-Verlag, Berlin—Heidelberg—New York, 1977] is devoted to the study of Banach lattices.

A partially ordered Banach space X over the reals is called a *Banach lattice* if the following conditions are satisfied:

(i) $x\leqq y$ implies $x+z\leqq y+z$, for every $x, y, z\in X$;

(ii) $ax\geqq 0$, for every $x\geqq 0$ in X and every non-negative real a;

(iii) for all $x, y\in X$ there exists a least upper bound $x\vee y$ and a greatest lower bound $x\wedge y$;

(iv) there exists a constant M such that $\|x\|\leqq M\,\|y\|$ whenever $|x|\leqq|y|$, where the absolute value $|x|$ of $x\in X$ is defined by $|x|=x\vee(-x)$.

The structure of Banach lattices is much simpler than that of general Banach spaces and their theory is therefore more complete and satisfactory. Many of the results concerning Banach lattices are not valid and sometimes even do not make sense for general Banach spaces. The theory of Banach lattices has many tools which are specific to this theory, in particular, the notions of p-convexity and p-concavity seem to be especially useful. These notions play a central role in the present volume and presumably will continue to dominate the theory of Banach lattices.

The book consists of two chapters, both subdividing into seven sections. The table of contents is quite detailed and gives a clear idea of the material discussed in each section. The basic standard theory of Banach lattices is contained is Sections 1 a)—c). The theory of p-convexity and p-concavity is presented in Sections 1 d)—f).

Chapter 2 is devoted to a detailed study of the structure of rearrangement invariant function spaces (r.i.f.s.) on [0, 1] and [0, ∞): a) Basic definitions, examples and results; b) The Boyd indices; c) The Haar and the trigonometric systems; d) Some results on complemented subspaces; e) Isomorphisms between r.i.f.s. and uniqueness of the r.i. structure; f) Applications of the Poisson process to r.i.f.s.

Three of the sections are concerned with the general theory of Banach spaces rather than with Banach lattices. Section 1 e) deals with the theory of uniform convexity, 1 g) with the approximation property, and 2 g) with geometric aspects of interpolation theory in general Banach spaces.

The prerequisities include, besides standard material from functional analysis and measure theory, only a superficial knowledge of the material presented in Volume I of this book. For the convenience of the reader the authors tried to discuss briefly in the appropriate places the notions and results from probability theory which they apply.

The overlap between this volume and existing books on lattice theory is small and consists mostly of the standard material presented in Sections 1 a)—b). The books of W. A. J. Luxemburg and A. C. Zaanen [*Riesz spaces* I, North—Holland, Amsterdam, 1971] and H. H. Schaefer [*Banach lattices and positive operators*, Springer-Verlag, Berlin—Heidelberg—New York, 1974] contain much additional material rather on vector lattices. The volume under review comprises the substantial progress made in the seventies.

To sum up, the present book is a rich and up-to-date account on this fast-growing and important subject. It is warmly recommended to everyone who wants to learn, or do research in, the theory of Banach spaces.

F. Móricz (Szeged)

M. Schreiber, Differential forms (A heuristic introduction), X+150 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

The theory of differential forms is one of the most frequently applied branches of mathematics not only in several fields of mathematics but also in theoretical physics. But the systematic treatment of differential forms requires an apparatus of topology and algebra which can be difficult for mathematicians and physicists working in other fields of research. The present book treats the theory of differential forms with minimal apparatus and very few prerequisites. The exposition is heuristic and concrete. A differential form is considered as a multi-dimensional integrand given on surfaces in Euclidean space, and the various operations (such as exterior derivation) are treated on an elementary level, from the geometrical point of view. Several formulas, such as Stokes'formula, are proved on such an elementary level as possible. The book contains a short introduction to integral geometry also.

It is addressed to mathematicians, physicists and students who are interested in a quick acquirement of differential forms techniques.

Z. I. Szabó (Szeged)

G. Takeuti and W. M. Zaring, Axiomatic Set Theory, V+238 pages, Springer-Verlag (Berlin—Heidelberg—New York, 1973).

This almost completely self-contained volume is a continuation of a previous one by the same authors ("Introduction to Axiomatic Set Theory", Springer-Verlag, 1971). The present book deals with three well-known methods for constructing models of the Zermelo—Fraenkel set theory: relative constructibility, Cohen's forcing and Boolean valued models. After developing Lévy-Schoenfield's theory of relative constructibility (Sections 7, 8, 9) a relationship is established between Cohen's technique of forcing (Sec. 10) and Scott-Solovay's theory of Boolean valued models (Sec. 13). In the first six sections some facts of Boolean algebras, Boolean σ-algebras, partial ordered structures, and topologies needed later on are collected. The remaining sections are devoted to a deeper investigation of the concepts introduced in the earlier sections.

The text is recommended for graduate students.

P. E.-Tóth (Szeged)

I. M. Yaglom, A simple non-euclidean geometry and its physical basis, XVIII+308 pages, Springer-Verlag (New York—Heidelberg—Berlin, 1979).

It is a hard problem of geometrical education to give a simple, relatively quick but deep synthetical treatment of classical non-euclidean geometries. The book of I. M. Yaglom proves that this program is realizable very elegantly from the mathematical point of view and the deep connections between these geometries and physics can also be illuminated on this level. This physical motivation of the classical geometries is the most important intrinsic value of the book.

Chapter I and II are simple but non-trivial introductions to plane Galilean geometry and to Galilean inversive geometry with plane and inversive Euclidean geometry. The next chapters treat the physical basis of Galilean geometry, the relativistic kinematic and relativistic Minkowskian geometry. At the end of the book the reader finds three supplements in which the author gives a systematical treatment of the nine plane geometries with their axiomatic characterization and analytic models.

The subject is accessible to anyone versed in elementary mathematics. The book is addressed mainly to students of mathematics, physics, and mathematical education.

Z. I. Szabó (Szeged)

Livres reçus par la rédaction

M. Aigner, Combinatorial theory (Grundlehren der mathematischen Wissenschaften — A Series of Comprehensive Studies in Mathematics, 234), VIII+483 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979. — DM 79,50.

Algebraic Topology, Proceedings of a Conference sponsored by the Canadian Mathematical Society, NSERC (Canada), and the University of Waterloo, June 1978. Edited by P. Hoffman and V. Snaith (Lecture Notes in Mathematics, Vol. 741), XI+655 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979. — DM 56,—.

Algebraic Topology, Proceedings of a Symposium held at Aarhus, Denmark, August 7—12, 1978. Edited by J. L. Dupont and I. H. Madsen (Lecture Notes in Mathematics, Vol. 763), VI+695 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979. — DM 60,—.

Analyse Harmonique sur les Groupes de Lie. II, Séminaire Nancy-Strasbourg 1976—1978. Edité par P. Eymard, J. Faraut, G. Schiffmann, R. Takahashi (Lecture Notes in Mathematics, Vol. 739), VI+646 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979. — DM 54,—.

B. Aupetit, Propriétés spectrales des algèbres de Banach (Lecture Notes in Mathematics, Vol. 735), XII+192 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979. — DM 25,—.